ing point," and thus the inter-focal distance of the real or imaginary foci, as the case may be, plainly cuts all the co-axial circles orthogonally.

*Corollary 8.* In the theorem, the hyperbola's asymptotes being also tangents, the circle through the points (say $e, f, e', f'$) in which any two tangents cut them, is also one of the co-axial system having the minor axis for its radical axis. So that $Ce.Cf=CS^2=Ce'.Cf'$.

## DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

NOTE ON SHORT METHODS IN ARITHMETICAL CALCULATIONS.

From time to time, we have sent to us for publication, "short cuts" and "lightning methods." Most of these are of no theoretical and little practical value, and hence cannot be given a place in the MONTHLY.

The short cuts below were sent to us by Mr. Charles H. Case of Chicago, who is now neariy 81 years old. He says the list, consisting of eighteen examples, was prepared for the students of Wheaton College, November, 1896. We publish a few of them because we have found some of them useful in practical computation, having used them for years.

Mr. Case says, "The examples given should be wrought without the use of more figures than are used in the same." The principles used may be found mainly in the algebraic formulae given below.

$$(a\div b)(b\div a)=1;\quad (a+b)^2=a^2+2ab+b^2;\quad (a+b)(a-b)=a^2-b^2).$$

1. $(5\frac{1}{2})^2=30\frac{1}{4}$; $(7\frac{1}{2})^2=56\frac{1}{4}$; $(65)^2=4225$; $(88)^2=7744$; $96^2=9216$; $36^2=1296$; $76^2=5776$.

2. $625^2=\begin{array}{r}360625\\ \underline{3\phantom{0625}}\\ 390625\end{array}$; $876^2=767376$; $2496^2=6230016$.

3. $$\begin{array}{rr}99649964125^2= & 9928129699281296015625\\ & +198562592\phantom{015625}\\ & +249124910\phantom{15625}\\ \hline & 9930115350113787015625\end{array}$$

4. $(2436)^2=\begin{array}{r}5761296\\ \underline{1728\phantom{00}}\\ 5934096\end{array}$; $68\times132=8976$; $8852\times8948=79207696$; $868\times932=808976$.

5. $48\frac{17}{29}\times 49\frac{12}{29}=2400\frac{697}{841}$; $1295\frac{57}{93}\times 1296\frac{36}{93}=1677615\frac{7343}{8649}$.

6. $7464\times 7536=56248704$; $88044\times 87956=7744000000-1936$.

7. $2777\frac{7}{9}\times 41666\frac{2}{3}\times 666\frac{2}{3}\times 54\times 24\times 52\times 7692307\frac{9}{13}\times 625\times 125\times 56\times 32\times 142857142857142 8\frac{4}{7}\times 2083\frac{1}{3}\times 48\times 833\frac{1}{3}\times 3125\times 68543764287590=\ldots$"

These and eleven other even longer computations are carried out mentally by Mr. Case.

The principle, $a^2=(a-b)(a+b)+b^2$, may be used in the squaring of any number, though it is not so readily used if the numbers consist of more than two digits. Thus,

$$87^2=(87-3)(87+3)+3^2=84\times 90+9,$$
$$92^2=(92-2)(92+2)+2^2=90\times 94+4.$$

This is the principle used in several of Mr. Case's calculutions. Thus, $(5\frac{1}{2})^2=(5\frac{1}{2}-\frac{1}{2})(5\frac{1}{2}+\frac{1}{2})+(\frac{1}{2})^2=30\frac{1}{4}$. ED. F.

**330. Proposed by R. D. CARMICHAEL, Princeton, N. J.**

An important function in the Theory of Numbers is one defined thus: $f(x)=1$ when $x>0$, $f(x)=0$ when $x=0$, $f(x)=-1$ when $x<0$. Two analytic expressions for $f(x)$ are the following:

$$f(x)=\lim_{n\doteq\infty} x^{1/(2n-1)},\ n=1,\ 2,\ \ldots;\quad f(x)=\lim_{n\doteq\infty}\frac{(x+1)^n-(x+1)^{-n}}{(x+1)^n+(x+1)^{-n}},\ x>-1.$$

It is required to find other non-trigonometric analytic expressions for this function. (There are several representations of $f(x)$ by means of trigonometric functions.)

**Remark by the PROPOSER.**

Professor F. H. Safford, of the University of Pennsylvania, has sent me the following expressions for the function defined in the problem:

$$\frac{2}{\pi}\int_0^\infty \frac{\sin xz}{z}dz,\quad \frac{2}{\pi}\int_0^\infty \frac{xdz}{x^2+z^2},\quad \lim_{m=+\infty}\frac{e^{xm}-e^{-xm}}{e^{xm}+e^{-xm}}.$$

**333. Proposed by R. D. CARMICHAEL, Princeton University.**

Sum the infinite series

$$\frac{1}{(m+1)^2}+\frac{(2m-1)}{(2m+1)^2}+\frac{(3m-1)^2}{(3m+1)^4}+\frac{(4m-1)^3}{(4m+1)^5}+\frac{(5m-1)^4}{(5m+1)^6}+\ldots$$

[No solution of this problem has been received.]

**334. Proposed by G. B. M. ZERR. A. M., Ph. D., Philadelphia, Pa.**

Sum the series, $2^n-n\,2^{n-2}+\frac{n(n-3)}{2!}2^{n-4}-\frac{n(n-4)(n-5)}{3!}2^{n-6}$
$+\frac{n(n-5)(n-6)(n-7)}{4!}2^{n-8}-\frac{n(n-6)(n-7)(n-8)(n-9)}{5!}2^{n-10}+\ldots$